# INVESTIGATION OF THE HYDRAULIC RESISTANCE IN POLYETHYLENE PIPELINES 

M. G. Sukharev, A. M. Karasevich,<br>\section*{R. V. Samoilov, and I. V. Tverskoi}

UDC 532.542.4

The hydraulic resistance to a flow propagating in polyethylene pipelines with a velocity characteristic of flows propagating in gas-supply systems has been experimentally investigated. The experimental data have been processes statistically with the use of the maximum-likehood method and regression analysis. The hydraulicresistance coefficient has been approximated using the Colebrook-White, Prandtl, and other models. Corrections to the hydraulic formulas used for calculating polyethylene pipelines in the process of their designing and service are proposed.

The study of the hydraulics of liquids and gases flowing in industrial pipelines is of great practical importance. The data fundamental for this field of research were obtained more than fifty years ago [1, 2]. There are also many reference books [3, 4] and textbooks containing information on this subject. However, the investigations carried out in the indicated field cannot be considered as completed. New publications devoted to various aspects of the pipeline hydraulics continue to appear [5-9]. The interest shown in this problem is explained by the following facts. The classical Nikuradse experiments [1, 2] were carried out with laminar, turbulent, and transient liquid flows propagating with different velocities. The data obtained for liquids were then extended to gases and supported by the experimental data on flows in industrial pipelines to a certain extent allowed by the apparatus used for measuring the parameters of the flows. More recent special investigations introduced certain corrections to the data obtained for flows at large Reynolds numbers [5] and for transient flows [6]. This has been possible due to the appearance of modern methods of measuring the parameters of flows with the use of computers.

In the above-indicated works, gas flows in pipelines with artificial [1, 2, 7, 8] and natural [1, 2, 7-9] roughness were investigated. We also know of works containing data on regimes of flow in industrial gas pipelines [9, 10]; however, these data were obtained for steel pipelines.

At present, polyethylene pipes are finding wider and wider application in gas-supply distribution systems. Polyethylene pipes have a number of advantages over the steel pipes: long service life (as long as 50 years), relatively low cost, high adaptability (suitability for inter-settlement, inner-town, and street service lines), and low inner-surface roughness that practically does not change with time. They are not prone to corrosion and, therefore, do not call for electrolytic protection. Polyethylene pipes will gradually supplant steel pipes in pipeline systems operating at a relatively low pressure. The foregoing points to the importance of development of models for simulation of flows in polyethylene pipelines.

To date, one and the same formulas have been used in technological calculations of steel and polyethylene pipelines. These formulas differed only in the numerical values of the roughness coefficient. Such extrapolation of hydraulic models of flows was done without sufficient substantiation, which can be given only by the experiment. However, we failed to find experimental data on the hydraulics of gas flows in polyethylene pipes and, therefore, decided to carry out a bench experiment.

Experimental. It is practically impossible to investigate gas flows in industrial polyethylene pipelines; therefore, we conducted experiments in the testing ground of the Scientific-Testing Center of the Central Institute of Aeroengine Engineering (STC CIAE). As the fluid, we used air and natural gas. Experiments were conducted in polyethylene pipes fabricated at the Moscow Gaztrubplast Plant. We first investigated flows in a "small" pipe of diameter

[^0]

Fig. 1. Basic diagram of placement of monomers in a pipeline.
32 mm and length 118.6 m with walls of thickness 3 mm and then flows in a "large" pipe of diameter 63 mm and length 854.89 m with walls of thickness 5.8 mm .

In the experiments conducted in the small pipe, the flow rate was determined with the use of a measuring diaphragm 25.1 mm in diameter and the pressure and differential pressure were measured by a standard manometer designed for $0-0.4 \mathrm{MPa}$ and by high-accuracy water piezometers of height 2 m . We measured the flow rate, pressure, and temperature of the gas at the input of the pipeline as well as the pressure of the gas at the output of the pipeline and in two of its intermediate cross sections, $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ (Fig. 1). The gas flow rate was determined in accordance with State Standards $8.563 .1-97$ and 8.563.2-97. The measurements were carried out on equipment of the STC CIAE.

The preliminary processing of the data obtained for the small pipe pointed to the lack of the metrological support of the experiment; therefore, more sophisticated measuring equipment (a microprocessor SuperFlow-IIE complex fabricated at the Sovtigaz Company) was used in the experiments conducted in the large pipe. The regime parameters measured in the process of experiments were stored in a magnetic medium.

Hereinafter we will concentrate our attention on the more reliable data obtained in the large pipe. The influence of the pipe bends, all things being equal, on the pressure loss was investigated in advance. The data obtained in the rectilinear pipe were compared with the data obtained in a twisted pipe having the maximum (allowable by its rigidity) number of rings. These data agree within the accuracy of the measuring equipment. Therefore, the placement of the large pipe was not varied: it formed five rings around a production building. We carried out two cycles of experiments in this pipe: one of the cycles was conducted with air, and the other cycle was conducted with natural gas. In each experiment, the flow rate and pressure of the gas at the input of the pipe was held, as far as possible, constant for several minutes. One regime was changed to the other by changing the position of gate valves mounted at the beginning and at the end of the pipeline. We tried to investigate the entire range of velocities and pressures characteristic of technological regimes in systems of natural-gas supply. The measurement data were processed with account for the fluctuations in the gas flow rate taking place within one experiment.

Model of Flow. The pressure loss in the gas flowing in a horizontal gas-supply distribution pipeline is calculated by the known semiempirical Darcy-Weisbach formula

$$
\begin{equation*}
p_{\mathrm{b}}^{2}-p_{\mathrm{e}}^{2}=0.1273 \cdot 10^{-13} \lambda L \rho z T D^{-5} q^{2} \tag{1}
\end{equation*}
$$

Relation (1) is written in a form suitable for calculating gas-supply systems operating at medium and high pressures. Modifications of this relation are also used in practice. The density $\rho$ and the flow rate $q$ of the gas are reduced to the "normal" ones at a temperature of $0^{\circ} \mathrm{C}$ and a pressure of 0.101 MPa . For actual gases, the coefficient $z$ is introduced into the Clapeyron formula as a correction. This coefficient is calculated at a medium pressure and a temperature $T$ by the Redlich-Kwong formula. The quantity $z$ changes insignificantly when passing from one regime to the other.

The hydraulic-resistance coefficient is represented as a function of the Reynolds number and the parameter $K_{\text {eq }}$ (equivalent roughness). There are a large number of approximations for the function $\lambda=\lambda\left(\mathrm{Re}, K_{\text {eq }}\right)$. Of them, the most well-known formulas are the Colebrook-White formula

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=-2 \log \left(\frac{K_{\mathrm{eq}} / D}{3.71}+\frac{2.51}{\operatorname{Re} \sqrt{\lambda}}\right) \tag{2}
\end{equation*}
$$

and the Prandtl formula for smooth pipes

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=2 \log \left(\frac{\operatorname{Re} \sqrt{\lambda}}{2.51}\right) \tag{3}
\end{equation*}
$$

Formula (3) is obtained from (2) when the second term in parentheses is much larger than the first term. In the case where the second term can be disregarded, (2) is rearranged into the Kármán formula:

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=2 \log \left(\frac{3.71 D}{K_{\mathrm{eq}}}\right) \tag{4}
\end{equation*}
$$

Different approximations are also used in practice. For example, in the industrial norms used for designing gas mains [11], the following formula is prescribed:

$$
\begin{equation*}
\lambda=0.067\left(\frac{158}{\operatorname{Re}}+\frac{2 K}{D}\right)^{0.2} \tag{5}
\end{equation*}
$$

The Al'tshul' formula is recommended for calculating gas-supply systems:

$$
\begin{equation*}
\lambda=0.11\left(\frac{68}{\operatorname{Re}}+\frac{K_{\mathrm{eq}}}{D}\right)^{0.25} \tag{6}
\end{equation*}
$$

This formula was eliminated from the last wording of the norms; however, it is used as before since other recommendations were not proposed.

The coefficient $K_{\text {eq }}$ characterizes a certain type of roughness, for example, the sand roughness used in the Nikuradse experiments. The values of $K_{\text {eq }}$ can differ for different batches of pipes used in industrial pipelines even in the case where the pipes included in these batches have equal diameters and are made of one and the same material [4]. Therefore, $K_{\text {eq }}$ should be considered as any general characteristic of the surface roughness. The former norms recommend using $K_{\mathrm{eq}}=0.01 \mathrm{~cm}$ for steel pipelines and $K_{\mathrm{eq}}=0.002 \mathrm{~cm}$ for polyethylene pipelines in formula (6). It is the practice to measure the surface roughness with the use of a roughness indicator and characterize it by a number of parameters. Even though the equivalent roughness $K_{\mathrm{eq}}$ is related to these parameters [8], it cannot be identified to any of them.

As is commonly supposed in the semiempirical approach, we will represent the quantity studied $\lambda$ as a function of the dimensionless arguments Re and $K_{\mathrm{eq}} / D$. In [12], relations (2)-(4) were obtained using models of two-dimensional axisymmetric flows having a boundary layer. The numerical values that agree most closely with the measurement data were used as coefficients in (2). In addition to relations (2)-(4), we used formulas (5) and (6), which are very close in essence to the Colebrook-White formula but are more simple and thus are widely used in practice.

Methods of Processing Measurement Data. The main problem of processing the measurement data is estimation of the coefficient $\lambda$ that is related to the measured parameters by relation (1). Since the measurement data contain errors and therefore represent variables, we estimated $\lambda$ from the mathematical statistics standpoint. The use of mathematical statistics must not be reduced to simple statistical procedures, such as calculation of means; it should include the selection of a processing method that would be adequate to the measured parameters and the supposed measurement error distributions. Under the conditions considered, the following set of measured parameters should be processes: $\Delta p, T_{0}, p_{0}, p_{1}, p_{2}$, and $p_{3}$. The quantities $p_{0}, \Delta p$, and $T_{0}$ are used for calculating the gas flow rate $q_{0}$ in the cross section $\mathrm{A}_{0}$. If the parameters of a gas flow do not change with time, the gas-flow rates in all the cross sections of a pipeline are equal: $q_{0}=q_{1}=q_{2}=q_{3}=q$. In this case, i.e., in the case where the flow is steady-state at each instant of time $t$ at which the parameters are measured, the set of parameters processed has the form

$$
p_{0 t}^{*}, p_{1 t}^{*}, p_{2 t}^{*}, p_{3 t}^{*}, q_{t}^{*}, T_{0 t}^{*}(t=1, \ldots, N) .
$$

The top asterisk denotes the measurement parameter. This set of measurements at a certain value of $t$ will be called the $t$ th series. A measured quantity is represented as the sum of its actual, but unknown, value and the measurement
error: $p^{*}=p+\delta p, q^{*}=q+\delta q$, and $T^{*}=T+\delta T$. The actual values of the parameters are determined from relation (1). At each instant of time $t$, the quantities $p_{j t}, q_{t}$, and $T_{t}$ are related by three independent relations. Having written relation (1) for pipeline sections $\left(\mathrm{A}_{0}-\mathrm{A}_{1}\right),\left(\mathrm{A}_{0}-\mathrm{A}_{2}\right)$, and $\left(\mathrm{A}_{0}-\mathrm{A}_{3}\right)$ with the use of the designations $u=p^{2}$ and $k=$ $0.1273 \cdot 10^{-13} \rho z T D^{-5}$, we obtain

$$
\begin{equation*}
u_{0}-u_{1}=\lambda L_{1} k q^{2}, u_{0}-u_{2}=\lambda L_{2} k q^{2}, u_{0}-u_{3}=\lambda L_{3} k q^{2} \tag{7}
\end{equation*}
$$

The factor $k$ has one and the same values in all three equalities since the temperature remains practically unchanged everywhere throughout the pipeline. Thus,

$$
\begin{equation*}
u_{0 t}-u_{j t}=\lambda L_{j} k q_{t}^{2} \quad(j=1,2,3 ; t=1, \ldots, N) \tag{8}
\end{equation*}
$$

The challenge of the statistical processing is estimation of the hydraulic-resistance coefficient by the measurement data. We will estimate this quantity using confluence analysis [13] and the maximum-likehood method. Let us write (8) in the form

$$
\begin{equation*}
\eta_{j}=v \xi \tag{9}
\end{equation*}
$$

where $\xi=q^{* 2}, \eta_{j}=u_{0}^{*}-u_{j} /\left(k L_{j}\right)$, and $v=\lambda(j=1,2,3)$.
In the subsequent discussion, we will assume that the distribution of the quantities $\xi$ and $\eta_{j}$ is normal: $\xi \sim N\left(x, \sigma_{\xi}^{2}\right)$ and $\eta_{j} \sim N\left(y_{j}, \sigma_{\eta_{j}}^{2}\right)$. The dispersions $D \xi=\sigma_{\xi}^{2}$ and $D \eta_{j}=\sigma_{\eta_{j}}^{2}$ are determined using the dispersions of the measurements

$$
\sigma_{\xi}^{2}=4 q^{2} \sigma_{q}^{2}, \quad \sigma_{\eta_{j}}^{2}=\frac{4}{k^{2} L_{j}^{2}}\left(u_{0}+u_{j}\right) \sigma_{p}^{2}
$$

The joint normal distribution of the random vector $\left(\xi, \eta_{1}, \eta_{2}, \eta_{3}\right)$ is characterized by the mathematical expectations and the covariance matrix $\mathbf{D}$ :

$$
\mathbf{D}=\frac{4 u_{0} \sigma_{p}^{2}}{k^{2}}\left\|\begin{array}{cccc}
\frac{k^{2}}{u_{0}} q^{2} \frac{\sigma_{q}^{2}}{\sigma_{p}^{2}} & 0 & 0 & 0 \\
0 & \frac{1}{L_{1}^{2}}\left(1+\frac{u_{1}}{u_{0}}\right) & \frac{1}{L_{1} L_{2}} & \frac{1}{L_{1} L_{3}} \\
0 & \frac{1}{L_{1} L_{2}} & \frac{1}{L_{2}^{2}}\left(1+\frac{u_{2}}{u_{0}}\right) & \frac{1}{L_{2} L_{3}} \\
0 & \frac{1}{L_{1} L_{3}} & \frac{1}{L_{2} L_{3}} & \frac{1}{L_{3}^{2}}\left(1+\frac{u_{3}}{u_{0}}\right)
\end{array}\right\|
$$

It is an easy matter to express the likelihood function, representing the density of distribution of a measurement as a random vector, in terms of the covariance matrix and obtain the desired estimates. In our case, it is an exponent whose power represents a quadratic function of the quantities $\xi_{t}$ and $\eta_{j t}$. Since the corresponding formulas are cumbersome, we will restrict our consideration to an approximation that, as the calculations have shown, gives quite satisfactory results. Since the diagonal elements are dominant in the $\mathbf{D}$ matrix, the nondiagonal elements can be disregarded in the first approximation, i.e., it may be assumed that $\operatorname{cov}\left(\eta_{j}, \eta_{k}\right)=0$ and $j \neq k$. In this case, the analytical representation of the likelihood function becomes much simpler:

$$
\operatorname{Lik} \sim\left(\sigma_{\xi} \sigma_{\eta_{1}} \sigma_{\eta_{2}} \sigma_{\eta_{3}}\right)^{N} \exp \left\{-\frac{1}{2 \sigma_{\xi}^{2}} \sum_{t=1}^{N}\left(\xi_{t}-x_{t}\right)^{2}-\sum_{j=1}^{3} \frac{1}{2 \sigma_{\eta_{j}}^{2}} \sum_{t=1}^{N}\left(\eta_{j t}-v x_{t}\right)^{2}\right\} .
$$

It is known [13] that, to correctly estimate the parameter $v$ in the structural relations (9), it is necessary to know the relation between the dispersions. Therefore, hereinafter the quantities

$$
\begin{equation*}
\sigma_{\eta_{j}}^{2} / \sigma_{\xi}^{2}=\mu_{j} \quad(j=1,2,3) \tag{10}
\end{equation*}
$$

will be assumed to be known. The maximum likelihood is estimated on the assumption that the function $\ln$ Lik is maximum:

$$
\begin{gather*}
7 \frac{\partial \ln \mathrm{Lik}}{\partial x_{t}} \equiv \frac{\xi_{t}-x_{t}}{\sigma_{\xi}^{2}}+v \sum_{j=1}^{3} \frac{\eta_{j t}-v x_{t}}{\mu_{j} \sigma_{\xi}^{2}}=0 \quad(t=1, \ldots, N), \\
\frac{\partial \ln \mathrm{Lik}}{\partial v} \equiv \sum_{j=1}^{3} \frac{\sum_{t=1}^{3}\left(\eta_{j t}-v x_{t}\right) x_{t}}{\mu_{j} \sigma_{\xi}^{2}}=0, \\
\frac{\partial \ln \mathrm{Lik}}{\partial \sigma_{\xi}} \equiv-\frac{4 N}{\sigma_{\xi}}+\frac{1}{\sigma_{\xi}^{3}}\left[\sum_{t=1}^{N}\left(\xi_{t}-x_{t}\right)^{2}+\sum_{j=1}^{3} \frac{1}{\mu_{j}} \sum_{t=1}^{N}\left(\eta_{j t}-v x_{t}\right)^{2}\right]=0 . \tag{11}
\end{gather*}
$$

System (11) was solved directly: $x_{t}=\left(\xi_{t}+c_{t} v\right) /\left(1+c v^{2}\right), c=\sum_{j=1}^{3} \mu_{j}^{-1}$, and $c_{t}=\sum_{j=1}^{3} \eta_{j t} / \mu_{j}$. It is an easy matter to derive a quadratic equation for determining the quantity $v$. The positive root is taken as a solution, and then all the other parameters are determined from system (11). The calculations have shown that, in our case, the results given by the simplified method are practically identical to the results given by a more exact method in which the assumption $\operatorname{cov}\left(\eta_{j}\right.$, $\left.\eta_{k}\right)=0$ is not used.

The quantities $\mu_{j}$ in (10) used in the calculations can be determined by replacing the dispersions in formulas (10) by the quantities estimated by a measurement, by the class of the apparatus accuracy, or by the possible range of change in the corresponding quantities determined a priori (in this case, a parametric investigation is carried out).

Along with the maximum-likelihood method, which is the most justified from the theoretical standpoint, we used the regression analysis (representing a particular case of the confluence analysis if the quantities $\mu_{j}$ are equal to zero or infinity) for processing the measurement data. Regression-analysis models include more simple formulas that give estimates differing insignificantly from the estimates given by more exact models.

As already noted, because of the fluctuations of a gas flow, the flow rate can be dependent on the space coordinate at a certain instant of time $t$. We have developed a method that allows one to take into account the change in the gas flow along a pipeline at each instant of time. It is known [14] that the following relations are true for a nonstationary gas flow in pipes:

$$
\begin{equation*}
u_{0 t}-u_{j t}=\lambda L_{j} k\left(\frac{q_{0 t}+q_{j t}}{2}\right)^{2} \quad(t=1, \ldots, N ; j=1,2,3) \tag{12}
\end{equation*}
$$

Let us now take $m$ successive measurements and assume that the flow is near-stationary at the initial and final instants of time. In this case, the material balance relations are fulfilled:


Fig. 2. Estimation of the coefficient $\lambda$ by combined data obtained in experiments with air and natural gas: a) estimation by system (11); b) estimation by formula (13); the solid curve corresponds to formula (6) at $K_{\mathrm{eq}}=0.002 \mathrm{~cm}$, the dashed line corresponds to the linear approximation at "small" Re and approximation (14) at "large" Re: 1) air; 2) natural gas.

$$
\sum_{t=1}^{m} q_{j t}=\sum_{t=1}^{m} q_{0 t}(j=1,2,3) .
$$

Using the regression analysis, we obtain

$$
\begin{equation*}
\sqrt{\lambda}=\frac{1}{3} \sum_{j=1}^{3}\left\langle\left(\frac{u_{0 t}-u_{j t}}{L_{j} k}\right)^{1 / 2}\right\rangle /\left\langle q_{0 t}\right\rangle \tag{13}
\end{equation*}
$$

The angle brackets denote the averaged values of the parameters used in a measurement. Formula (13) was derived on the assumption that $\left\langle\left(q_{0 t}+q_{j t}\right) / 2\right\rangle=\left\langle q_{0}\right\rangle$.

Analysis of Results. Figure 2 shows the hydraulic-resistance coefficients determined by Eqs. (11) (considered as the most exact model of processing of the measurement data that accounts for the correlation relations between the quantities) and formula (13). It is seen even visually that the scatter of the data on the correlation field is smaller in the case where formula (13) is used. This points to the fact that taking into account the fluctuations of the gas-flow rate significantly influences the results of the processing.

It is also seen from Fig. 2 that, at $K_{\text {eq }}=0.002 \mathrm{~cm}$, recommended by the norms, model (6) approximates the experimental data inadequately. The largest differences are obtained at small and large values of the Reynolds number. Analogous conclusions were made for models (5) and (2). Model (4), which does not take into account the dependence of $\lambda$ on Re, was found to be unsuitable for interpretation of experimental data and, therefore, was excluded from the further consideration.


Fig. 3. Estimation of the equivalent roughness by formula (6) (processing by model (13)): 1) air; 2) natural gas. $K_{\text {eq }}, \mathrm{cm}$.

Figure 3 shows the equivalent roughness values calculated using $\lambda$ by formula (6). If model (6) defined the flow in polyethylene pipes adequately, the roughness estimated as any physical parameter would be independent of the Reynolds number. However, it is evident from the figure that the estimated value of $K_{\text {eq }}$ decreases with increase in Re. Analogous conclusions were drawn for models (2) and (5).

At small values of Re, models (6) and (2) give close values of $K_{\text {eq }}$, while model (5) gives much larger values of this parameter. This is explained by the fact that formula (5) was derived for gas mains, i.e., for pipes of large diameters and high gas-flow rates. Therefore, it is reasonable that this formula is little suitable for estimating the experimental data obtained at relatively small values of Re .

Thus, the above-described models of flow are inadequate and should be refined. We have performed a series of calculations on selection of parameters of various formulas defining the dependence $\lambda=\lambda\left(\operatorname{Re}, K_{\text {eq }}\right)$. In this case, the quantity $K_{\mathrm{eq}}$ was prescribed or was a parameter estimated in the process of approximation. Below, we present some of the results obtained. Let us first consider the approximation

$$
\begin{equation*}
\lambda=0.11\left(\frac{a}{\mathrm{Re}}+\frac{K_{\mathrm{eq}}}{D}\right)^{n} \tag{14}
\end{equation*}
$$

The parameters $a$ and $n$ were estimated by the method of least squares:

$$
\begin{equation*}
\hat{R}_{N}^{2}=\sum_{i=1}^{N}\left[\lambda_{i}-0.11\left(\frac{a}{\operatorname{Re}_{i}}+\frac{K_{\mathrm{eq}}}{D}\right)^{n}\right]^{2} \rightarrow \min _{a, n} \tag{15}
\end{equation*}
$$

The initial data were correlation-field points (Fig. 2). Two values of the equivalent roughness were used: $K_{\mathrm{eq}}=0.002 \mathrm{~cm}$ was recommended by the former norms for polyethylene pipes and $K_{\mathrm{eq}}=0.00185 \mathrm{~cm}$ was determined by the results of a special profilometry of the inner surface of pipes.

The results presented in Table 1 additionally point to the fact that formula (13) offers advantages over the other formulas in estimating the coefficient $\lambda$. The quantity $\hat{\sigma}_{N}^{2}=\hat{R}_{N}^{2} /(N-2)$ calculated by the residual sum of the squares $\hat{R}_{N}^{2}$ characterizes the quality of the approximation. The last row contains estimates of the measurement data obtained for the combined fluid in the large pipe. Rounding the estimates to $n=0.3$ and $a=300$, we obtain a formula suitable for practice that gives a value of $\hat{\sigma}_{N}^{2}$ differing insignificantly from the mathematical optimum. The above-indicated data were obtained by processing the measurement data obtained at Reynolds numbers $\operatorname{Re} \geq 60,000$. It is seen from Fig. 2b that, at smaller values of Re, the character of the dependence $\lambda=\lambda$ (ln Re) changes. The dashed line in Fig. 2b corresponds to an interval by interval approximation in which a linear function was used for "small" Re.

Similar results were obtained for the Colebrook-White formula (2) into which the parameters $a$ and $n$ were introduced:

TABLE 1. Parameters and Residual Sum of Squares Determined by Approximation (14)

| Method of <br> estimation $\lambda$ | Fluid | $n$ | $a$ | $\hat{\sigma}_{N}^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $(11)$ | Air | 0.291 | 267.68 | $5.589 \cdot 10^{-7}$ |
| $(13)$ | " | 0.304 | 358.01 | $2.968 \cdot 10^{-7}$ |
| $(11)$ | Natural gas | 0.289 | 238.81 | $1.298 \cdot 10^{-6}$ |
| $(13)$ | » | 0.284 | 217.93 | $4.304 \cdot 10^{-7}$ |
| $(13)$ | Air and natural gas | 0.296 | 297.19 | $3.830 \cdot 10^{-7}$ |



Fig. 4. Approximation of the dependence $\lambda\left(\mathrm{Re}, K_{\mathrm{eq}}\right)$ : the solid curve corresponds to formula (14) at $K_{\text {eq }}=0.002 \mathrm{~cm}, n=0.3$, and $a=300$; the dashed curve corresponds to formula (14) at $K_{\mathrm{eq}}=0, n=0.2617$, and $a=160.95$ [1) air; 2) natural gas].

$$
\begin{equation*}
\sqrt{\frac{1}{\lambda}}=-n \log \left(\frac{a}{\operatorname{Re} \sqrt{\lambda}}+\frac{K_{\mathrm{eq}}}{3.7 D}\right) . \tag{16}
\end{equation*}
$$

As the initial data, we used the estimates of $\lambda$ made by formula (13). For the combined fluid, $n=2.4935, a=20.53$, and $\hat{\sigma}_{N}^{2}=0.01168$.

The question arises of whether there is a difference between the hydraulic-resistance coefficients to the air and natural-gas flows in one and the same pipe. It may be suggested that the physical nature of the turbulent processes occurring in the boundary layer of a methane flow in polyethylene pipes differs from that of other fluids flowing in these pipes. This potentially possible phenomenon can be explained by the fact that the structures of the pipe and fluid are identical at the molecular level.

Judging by the correlation field shown in Fig. 2, the difference between the values of $\lambda$ for methane and air is small, even though the value of $\lambda$ for methane is somewhat smaller than that for air. The same conclusions were obtained by statistical methods. We now consider the deviations of the experimental data from the approximation curve. Let us divide them into two groups: $\lambda_{11}, \ldots, \lambda_{1 k}$ and $\lambda_{21}, \ldots, \lambda_{2 m}$. The first group was obtained for air and the second group was obtained for methane. The hypothesis made above will be sustained if the data in the second group are much smaller than the data in the first group. The existence or absence of differences between these groups was determined by the Wilcoxon criterion [15]. We calculated the sum of the ranks of the quantities comprising the second group and compared them with the critical value. In our case ( $k=18, m=17$ ), the critical value is equal to 255 at a significance level of $\alpha=0.05$ and 266 at $\alpha=0.1$. The criterion, i.e., the sum of the ranks, was 267.5 in the case where the data were processed using model (6) and 266 in the case where the data were processed using model (2). Thus, we neither proved nor ruled out the difference between the hydraulic resistances of polyethylene pipes to the methane and air flowing in them. This phenomenon should be specially investigated.

In the next series of approximations, the equivalent-roughness coefficient $K_{\text {eq }}$ was estimated. We investigated relation (6), where $K_{\mathrm{eq}}$ and $n$ were considered as parameters. This approximation was found to be less appropriate than (14).

We now consider the popular notion of "hydraulically smooth pipe" that can be naturally used for polyethylene pipes. The measurement data were approximated by formula (14) with $K_{\text {eq }}=0$. For the combined fluid we obtained $a=160.95, n=0.2617$, and $\sigma_{N}^{2}=4.061 \cdot 10^{-7}$.

As is seen, the remaining sums of squares are fairly close to the analogous values presented in Table 1. As is seen from Fig. 4, the corresponding curves are close. Thus, the model of hydraulically smooth pipes gives data close to the corresponding experimental data. We recommend using formula (14) for calculations at $K_{\text {eq }}=0, a=80$, and $n=0.237$ or (with much the same approximation error) $K_{\text {eq }}=0, a=375$, and $n=0.3$.

## CONCLUSIONS

1. The hydraulic resistance of pipelines to the flows considered depends substantially on the Reynolds number Re. Therefore, the coefficient $\lambda$ should be determined using functional dependences analogous to the Colebrook-White model (2) and the Altshuler model (6).
2. The Colebrook-White and Al'tshul' models with the adopted numerical values of the coefficients, which adequately define the hydraulic resistance to the flows in steel, reinforced-concrete, and other pipelines, cannot be considered as sufficiently exact for the gas flows in polyethylene pipelines. We failed to relate the parameter $K_{\text {eq }}$ entering into these models to the coefficient of physical roughness of the inner surface of a pipe.
3. It was found that the experimental data are well approximated by relation (14). If $K_{\text {eq }}=0.002 \mathrm{~cm}$, the best approximation is obtained at $a=300$ and $n=0.30$. This formula can be recommended for designing the gas-supply systems in which polyethylene pipes are used.
4. The model of hydraulically smooth pipes ( $K_{\mathrm{eq}}=0$ ) gives a good approximation. It can also be recommended for practical calculations at $a=80$ and $n=0.237$.
5. The validity of introducing the technical-roughness coefficients into calculation models should be verified in experiments with pipes of different producers.

The authors express their thanks to Prof. V. M. Entov for valuable observations concerning the present work.

## NOTATION

$a$, approximation parameter in formulas (14) and (16); c $=\sum_{j=1}^{3} \mu_{j}^{-1}, c_{t}=\sum_{j=1}^{3} \eta_{j t} / \mu_{j}$, designations used in the
solution of Eqs. (11); $D$, inner diameter of the pipe, $\mathrm{m} ; \mathbf{D}$, covariance matrix; $K$, roughness coefficient in the formula used for calculating the hydraulic resistance of gas mains; $K_{\text {eq }}$, equivalent-roughness coefficient in the formula used for calculating the hydraulic resistance of gas pipelines in the gas-supply distribution systems; $k$, coefficient remaining unchanged in each series of experiments $\left(k=0.1273 \cdot 10^{-13} \rho z T D^{-5}\right) ; L$, length of the pipeline, $\mathrm{m} ; l_{1}, l_{2}, l_{3}$, length of the pipeline sections (Fig. 1), m; $L=l_{1}, L_{2}=l_{1}+l_{2}, L_{3}=l_{1}+l_{2}+l_{3} ; M$, mathematical expectation symbol; $N$, number of instants the parameters are measured; $n$, approximation parameters in formulas (14) and (16); $p_{\mathrm{b}}$ and $p_{\mathrm{e}}$, absolute pressures of the gas at the beginning and end of the pipeline averaged over its cross section, MPa; $p_{0}-p_{3}$, pressure of the gas in cross sections $\mathrm{A}_{0}-\mathrm{A}_{3}$ (Fig. 1), $\mathrm{MPa} ; p_{0 t}^{*}, p_{1 t}^{*}, p_{2 t}^{*}, p_{3 t}^{*}$, measured values of the pressure, MPa; $q$, gas-flow rate, $\mathrm{m}^{3} / \mathrm{h} ; q_{0}-q_{3}$, flow rate of the gas in cross sections $\mathrm{A}_{0}-\mathrm{A}_{3}$ (Fig. 1 ), $\mathrm{m}^{3} / \mathrm{h} ; q_{t}^{*}$, measured values of the gas-flow rate, $\mathrm{m}^{3} / \mathrm{h} ; R_{N}^{2}$, minimum value of criterion (15); Re, Reynolds number; $T$, average temperature, K ; $T_{0}$, temperature of the gas in the initial cross section, $\mathrm{K} ; T_{0 t}^{*}$, measured values of the temperature, $\mathrm{K} ; u=p^{2}$, potential, $\mathrm{MPa}^{2} ; x=M \xi$, mathematical expectation of $\xi ; y_{j}=M \eta_{j}$, mathematical expectation of $\eta_{j} ; z$, compressibility coefficient; $\alpha$, significance level; $\Delta p$, differential pressure at the diaphragm positioned in the initial cross section $\mathrm{A}_{0}$ of the pipe-
line (Fig. 1), MPa; $\delta p$, error in measuring the pressure, MPa; $\delta q$, error in measuring the gas-flow rate, $\mathrm{m}^{3} / \mathrm{h}$; $\delta T$, error in measuring the temperature, K ; $\lambda$, hydraulic-resistance coefficient; $\mu_{j}$, ratio between the dispersions (10); $v$, proportionality coefficient in the model of statistical processing (9); $\rho$, gas density, $\mathrm{kg} / \mathrm{m}^{3} ; \sigma_{p}^{2}$, dispersion in measuring the pressure; $\sigma_{q}^{2}$, dispersion in measuring the gas-flow rate; $\sigma_{\xi}^{2}=D \xi$, dispersion of the quantity $\xi ; \sigma_{\eta_{j}}^{2}=D \eta_{j}$, dispersion of the quantity $\eta_{j} ; \hat{\sigma}_{N}^{2}$, estimate of the approximation accuracy; $\xi$, random quantity, function of the parameters measured (see formula (9)); $\eta_{j}$, random quantities, functions of the parameters measured (see formula (9)). Subscripts: b, beginning of a section; e, end of a section; eq; equivalent (roughness); $i$, number of measurement; $j$, quantity belonging in section $\mathrm{A}_{j-1}-\mathrm{A}_{j}$ or section $\mathrm{A}_{j}$ (Fig. 1); $t$, instant of time; 0, 1, 2, 3, quantity belonging in cross sections $0-3$ respectively (Fig. 1).

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[^0]:    Promgaz Company, 6 Nametkin Str., Moscow, 117420, Russia; email: M.Sukharev@ promgaz.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 2, pp. 136-144, March-April, 2005. Original article submitted December 2, 2003.

